# **Voronoi Treemaps** An introduction & explanation.

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### What are Voronoi diagrams ?





Constructed and captured by Christopher Oser using <u>Alex Beutel's</u> Interactive Voronoi Diagram Generator.

### **Basic Theory**

- Set S of n sites, whereby  $n \ge 3$ .
- Sites are on an Euclidean plane.
- Euclidean distance between two points, a and b, given as:

$$d(a,b) = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}$$

• Bisector B(a,b) is the locus of all points at equal distance of a and b.



# Voronoi Region

- The bisector distinguishes the half plane:  $H(a,b) = \{x \mid d(a,x) \le d(b,x)\}$
- The Voronoi region is the intersection of all n-1 half planes H(a,x), where x ranges over all other points in the set S.
- So the Voronoi region V(a,S) contains all points closest to a in respect to the set S.



# Some more definitions

- The boundary between two Voronoi regions is a Voronoi edge.
- Endpoints of Voronoi edges are called Voronoi vertices.



# **Delaunay Triangulation**

- Delaunay Tessellation is constructed by connecting any two points a,b of S, for which there is a circle that passes through them but contains no other points in S.
- If S in general position ⇒ tessellation turns into triangulation.
- Delaunay Tessellation/Triangulation is the dual of Voronoi diagram.



# **Delaunay Triangulation**

- Delaunay edge  $\Leftrightarrow$  Voronoi edge
- Delaunay face ⇔ Voronoi vertex
- Delaunay vertex ⇔ Voronoi region
- Duality useful in many algorithms.
- If you have one, you can get the other.



# **Construction Algorithms**

- Incremental:
  - Points are added one at a time.
  - In every step conflicts are analyzed & diagram is adapted.
  - $\circ$  Time: O(n<sup>2</sup>) worst case
  - BUT randomized/expected: O(n log n)
- Divide & Conquer:
  - S is split in two equal sets  $\Rightarrow$  L and R.
  - Recursively split sub-sets until only three or two points left.
  - Merge sub-diagrams to the final Voronoi diagram.
  - $\circ$  Time: O(n log n)

# **Construction Algorithms**

- Plane Sweep (Fortune's Algorithm):
  - Vertical line sweeps over plane.
  - Points are stored in y-order.
  - Voronoi regions are updated accordingly.
  - Time: O(n log n)
- Lift to 3-space
  - Project plane to 3rd dimension. ( $z = x^2 + y^2$ )
  - Use existing algorithms to compute convex hulls of n points.
  - Translate results back to 2 dimensional space.
  - Time: O(n log n)



# Voronoi Diagram Implementations

#### Incremental:

 LEDA [C++] https://www8.cs.umu.se/kurser/TDBAfl/VT06/algorithms/WEBSITE/IMPLEMEN/LEDA/IMPLEMEN.HTM
 Khuyen Tran [Python] https://github.com/khuventran1401/Voronoi-diagram

#### Divide & Conquer:

Alex Shavlovsky [Java]
 Ittps://dithub.com/alexshavlovsky/VoronolDiagram.JavaRecursive

#### Sweep-line:

- Mathias Westerdahl [Java]
- Raymond Hill [JS]
  https://github.com/gorhill/Javascript-Voronoi
- D3-voronoi [JS library]

#### Lift to 3-space:

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• <u>Qhull</u> [C]

# Applications

- Computer Science:
  - Computing Textures
  - Classification
  - Clustering
  - Localization (closest point)
- Biology: Modelling structures.
- Chemistry: Computing atomic charges.
- Geometry: Nearest neighbor queries.
- Meteorology: Analyzing spatially distributed data.

# Weighted Voronoi Diagrams

- In DataVis we want to weight visualizations according to data.
- Weights are assigned to the points in S.
- Weights influence size of Voronoi region.
- Makes it possible to customize size:
  - Bigger weight  $\Rightarrow$  larger region
  - Smaller weight  $\Rightarrow$  smaller region
  - All other properties remain the same
- Usually computed using power diagrams. But also:
  - Additively weighted
  - Multiplicatively weighted



# **Power Diagram**

- Voronoi-like
- Uses Circles/Spheres to define area of influence:

Radius  $r = \sqrt{w(p)}$ 

• Power Function:

 $pow(x,p) = (x - p)^T (x - p) - w(p)$ 

• Intersection of areas result in diagram.



Constructed and captured by Christopher Oser using GeoGebra.

# Other weighting methods

- Additively:
  - o add\_distance(x, p) = distance(x, p) weight

- Multiplicatively:
  - o mul\_distance(x, p) = distance(x, p) / weight
- Both yield curved diagrams, which is why we focus on power diagrams.

### Treemaps

- Method does not originate in Voronoi diagrams.
- Used to display hierarchical data.
- Nested polygons (usually rectangles) visualize the hierarchical nature of the data.
- Can be weighted by additional attributed data.
- <u>Treemap</u> Software developed by Ben Shneiderman's team at University of Maryland.

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# Weighted Voronoi Treemaps

- Use tree mapping to visualize hierarchical data.
  - Voronoi diagram for each layer
  - Embed child diagrams in parent Voronoi region
- Instead of rectangles  $\Rightarrow$  Voronoi regions.
- Assign weights:
  - Arbitrarily, for customization
  - According to children



Constructed and captured by Christopher Oser using <u>IVT</u>.



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# History of Voronoi Treemaps

• First introduced: InfoSky by Andrews et al, 2002.

https://doi.org/10.1057/PALGRAVE.IVS.9500023

- Used in visual explorer.
- Large hierarchical datasets visualized in voronoi diagram.

• Also thoroughly dealt with in <u>"Voronoi Treemaps" by Balzer</u>, 2005.

https://doi.org/10.1109/INFVIS.2005.1532128

- Establishes overview.
- Addresses theory in detail.
- Christened the technique

# Voronoi Treemap Libraries



https://github.com/Kcnarf/d3-voronoi-treemap

<u>ArlindNocaj Voronoi-Treemap-Library</u> [Java]

https://github.com/ArlindNocaj/Voronoi-Treemap-Library

Voronoi Treemaps in R - Paul Murrell [R]

https://www.stat.auckland.ac.nz/~paul/Reports/VoronoiTreemap/voronoiTreeMap.html

• Tableau template by Tristan Guillevin [Tableau]

https://ladataviz.com/2020/01/02/build-a-voronoi-treemap-in-tableau-in-two-steps/

# **Treemap Applications**

- FoamTree
  - Commercial software
  - Not open source
  - Showcase video https://youtu.be/-82bKzipfTl



Constructed and captured by Christopher Oser using FoamTree.



#### • IVT

- Developed at TU Graz
- Oser, Ruplitsch, Weissl, Gruber
- Open source
- Uses d3-voronoi-treemap (for now)
- Showcase video https://youtu.be/phWakUsk-7Y

# Conclusion

- Voronoi Diagram introduction.
- Focus on Voronoi Treemaps.
- Visualizing hierarchical data.
- Interactive visualizations are desirable.
- Further work on IVT to come (Master Thesis).